

# Entanglement induced by a two-mode thermal field

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The entanglement between two identical two-level atoms interacting with two mode thermal field through a nondegenerate two-photon process has been suggested. It has been shown that for some atomic initial state the entanglement induced by nondegenerate two-photon interaction is larger than that induced by one-photon and degenerate two-photon processes.

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The quantum entanglement plays a key role in quantum information and quantum computation [1]. Real quantum systems will inevitably be influenced by surrounding environments. Although the interaction between the environment and quantum systems can lead to decoherence, it can also induce entanglement. Recently, S.Bose et al. [2] have shown that entanglement can always arise in the interaction of an arbitrary large system in any mixed state with a single qubit in a pure state, and illustrated this using the Jaynes-Cummings interaction of a two-level atom in a pure state with a field in a thermal state at an arbitrary high temperature. M.Kim et al. [3] have investigated the entanglement of two identical two-level atoms with one-photon transition induced by a single-mode thermal field. L.Zhou et al [4] have considered the same problem for nonidentical atoms. The entanglement of two identical two-level atoms through nonlinear two-photon interaction with one-mode thermal field has been studied by L.Zhou et al. [5]. They have shown that atom-atom entanglement induced by nonlinear interaction is larger than that induced by linear interaction. It is of interest to investigate the entanglement between two identical atoms induced by thermal noise through other nonlinear processes. In this paper we dwell on two-photon nondegenerate interaction of two atoms with two mode of thermal field. The one-atom models with nondegenerate two-photon interaction in cavity have attracted much attention beginning with the paper of S.Gou [6]. We have obtained that for some atomic initial condition the entanglement induced by nondegenerate interaction may be larger than that induced by degenerate two-photon process.

Let us consider the system of two identical two-level atoms with frequency of the atomic transition  $\omega_0$  interacting with two modes of quantum electromagnetic field with frequencies  $\omega_1$  and  $\omega_2$  through a nondegenerate two-photon process. For simplicity we ignore the Stark shift and assume the lossless cavity modes to be at two-photon resonance with the atomic transition, i.e. the condition  $\omega_1 + \omega_2 = \omega_0$  takes place. Then, the Hamiltonian of the considered system in interaction picture and RWA approximation can be written in the form:

$$H_I = \hbar g \sum_{i=1}^2 (a_1^+ a_2^+ R_i^- + R_i^+ a_1 a_2), \quad (1)$$

where  $a_j^+$  and  $a_j$  are the creation and annihilation operators of photons of  $j$ th cavity mode ( $j = 1, 2$ ),  $R_i^+$  and  $R_i^-$  are the raising and the lowering operators for the  $i$ th atom,  $g$  is the coupling constant between the atom and the cavity field.

We denote by  $|+\rangle$  and  $|-\rangle$  the excited and ground states of a single two-level atom and by  $|n\rangle$  the Fock state of the electromagnetic field. The two-atom wave function can be expressed as a combination of state vectors of the form  $|v_1, v_2\rangle = |v_1\rangle |v_2\rangle$ , where  $v_1, v_2 = +, -$ .

The density operator for the atom-field system follows a unitary time evolution generated by the evolution operator  $U(t) = \exp(-iHt/\hbar)$ . In the two-atom basis  $|+, +\rangle, |+, -\rangle, |-, +\rangle, |-, -\rangle$  the analytical form of the evolution operator  $U(t)$  is given by

$$U(t) = \begin{pmatrix} a_1 a_2 C a_1^+ a_2^+ + 1 & -i a_1 a_2 S & -i a_1 a_2 S & a_1 a_2 C a_1 a_2 \\ -i a_1^+ a_2^+ S & \frac{1}{2}(\cos \Omega t + 1) & \frac{1}{2}(\cos \Omega t - 1) & -i S a_1 a_2 \\ -i a_1^+ a_2^+ S & \frac{1}{2}(\cos \Omega t - 1) & \frac{1}{2}(\cos \Omega t + 1) & -i S a_1 a_2 \\ a_1^+ a_2^+ C a_1^+ a_2^+ & -i a_1^+ a_2^+ S & -i a_1^+ a_2^+ S & a_1^+ a_2^+ C a_1 a_2 + 1 \end{pmatrix}, \quad (2)$$

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where

$$\Omega = g \sqrt{2(a_1 a_2 a_1^+ a_2^+ + a_1^+ a_2^+ a_1 a_2)}$$

and

$$C = \frac{2g^2}{\Omega^2} (\cos \Omega t - 1), \quad S = \frac{g}{\Omega} \sin \Omega t.$$

Let the cavity field is initially in two-mode thermal state with mean photon numbers  $\bar{n}_1$  and  $\bar{n}_2$ . The thermal radiation field is a weighted mixture of Fock states for each modes and its density operator is represented by

$$\rho_F(0) = \sum_{n_1 n_2} p_{n_1} p_{n_2} |n_1\rangle\langle n_1| |n_2\rangle\langle n_2|,$$

where the weight functions  $p_{n_i}$  is

$$p_{n_i} = \frac{\bar{n}_i^{n_i}}{(1 + \bar{n}_i)^{n_i+1}} \quad (i = 1, 2)$$

and the mean photon number  $\bar{n}_i$  in the  $i$ th cavity mode ( $i = 1, 2$ ) is  $\bar{n}_i = (\exp(\hbar\omega_i/k_B T) - 1)^{-1}$ , where  $T$  is the equilibrium cavity temperature and  $k_B$  is the Boltzmann constant.

To investigate the entanglement between atoms one can obtain the time-dependent reduced atomic density operator by means of the combined atom-field density operator over the field variables:

$$\rho_A(t) = Tr_F U(t) \rho(0) U^+(t). \quad (3)$$

If the atoms are initially in a pure state such as  $|+, +\rangle$ ,  $|+, -\rangle$ ,  $|-, +\rangle$ , or  $|-, -\rangle$ , the atomic density operator with using formulae (2), (3) may be expressed in a similar way to the nondegenerate two-photon case as has been obtained earlier by L.Zhou et al. [5]

$$\rho_A(t) = \begin{pmatrix} A & 0 & 0 & 0 \\ 0 & B & E & 0 \\ 0 & E & C & 0 \\ 0 & 0 & 0 & D \end{pmatrix}. \quad (4)$$

For two-qubit system described by density operator  $\rho_A(t)$ , a measure of entanglement can be defined in terms of the negative eigenvalues of partial transposition [6], [7]

$$\varepsilon = -2 \sum_i \mu_i^-,$$

where  $\mu_i^-$  are the negative eigenvalues of the partial transposition of  $\rho_A(t)$ . When  $\varepsilon = 0$  the two atoms are separable and  $\varepsilon > 0$  means the atom-atom entanglement. The case  $\varepsilon = 1$  indicates maximum entanglement. For partial transposition of (4) we have only one eigenvalue which can be negative. This value is  $\mu_1^- = \frac{1}{2}(D + A - \sqrt{(D - A)^2 + 4E^2})$ . The eigenvalue  $\mu_1^-$  becomes negative if and only if  $E > \sqrt{AD}$ . Under this condition, the measure of entanglement is

$$\varepsilon = \sqrt{(D - A)^2 + 4E^2} - D - A. \quad (5)$$

Let us consider the time-dependent measure of entanglement for various initial pure states of the atomic subsystem.

1. Let atoms are initially in the state  $|+, -\rangle$ . Then, the matrix elements of the atomic reduced density operator (4) are

$$\begin{aligned} A &= \sum_{n_1 n_2} p_{n_1} p_{n_2} n_1 n_2 S_{n_1, n_2}^2, \\ B &= \sum_{n_1 n_2} p_{n_1} p_{n_2} \cos^4 \left( \frac{\Omega_{n_1, n_2} t}{2} \right), \\ C &= \sum_{n_1 n_2} p_{n_1} p_{n_2} \sin^4 \left( \frac{\Omega_{n_1, n_2} t}{2} \right), \\ D &= \sum_{n_1 n_2} p_{n_1} p_{n_2} (n_1 + 1)(n_2 + 1) S_{n_1, n_2}^2, \\ E &= -\frac{1}{4} \sum_{n_1 n_2} p_{n_1} p_{n_2} \sin^2 (\Omega_{n_1, n_2} t), \end{aligned} \quad (6)$$

where

$$O_{n_1, n_2} = \langle n_1 | \langle n_2 | O | n_1 \rangle | n_2 \rangle.$$

The results of numerical calculations of entanglement on the basis of the formulae (5) and (6) are shown in Fig.1. In this the dependencies of the Peres-Horodecki parameter  $\varepsilon$  versus dimensionless time  $gt$  is presented for models with  $\bar{n}_1 = \bar{n}_2 = 0.3$  (dashed line) and  $\bar{n}_1 = \bar{n}_2 = 1$  (solid line). Comparing curves with that for one-photon [3] and degenerate two-photon [5] interaction, we find that contrary to above mentioned models the entanglement for considered model takes place for all instances only for sufficiently large intensities of the input cavity modes. The entanglement through nondegenerate two-photon interaction reveals the nonlinear behaviour in a similar way to the degenerate interaction but its value is less than that for aforementioned model with same values of input intensity.

2. If both atoms are initially in the ground state, i.e. the atomic state is  $| -, - \rangle$ , then, the matrix elements of the atomic density operator (4) are

$$\begin{aligned} A &= \sum_{n_1 n_2} p_{n_1} p_{n_2} n_1 (n_1 - 1) n_2 (n_2 - 1) C_{n_1-1, n_2-1}^2, \\ B = C = E &= \sum_{n_1 n_2} p_{n_1} p_{n_2} n_1 n_2 S_{n_1-1, n_2-1}^2, \\ D &= -\frac{1}{4} \sum_{n_1 n_2} p_{n_1} p_{n_2} [n_1 n_2 C_{n_1-1, n_2-1} + 1]^2. \end{aligned} \quad (7)$$

The time evolution of the entanglement in this case is shown in Fig.2 for the same values of input intensities of the cavity modes. We observe again the difference between considered and degenerate two-photon process. If two atoms initially both in ground state, they only become slightly entangled through degenerate two-photon processes [5] but for nondegenerate interaction the value of Peres-Horodecki parameter  $\varepsilon$  compares favourably with that for previous case when atoms are initially in the state  $| +, - \rangle$ . With same input intensities for considered model the maximum values of  $\varepsilon$  are much larger than those for degenerate two-photon model.

3. For both atoms in the initial excited state  $| +, + \rangle$  we have

$$\begin{aligned} A &= \sum_{n_1 n_2} p_{n_1} p_{n_2} [(n_1 + 1)(n_2 + 1) C_{n_1+1, n_2+1}^2 + 1]^2, \\ B = C = E &= \sum_{n_1 n_2} p_{n_1} p_{n_2} (n_1 + 1)(n_2 + 1) S_{n_1+1, n_2+1}^2, \\ D &= \sum_{n_1 n_2} p_{n_1} p_{n_2} (n_1 + 1)(n_1 + 2)(n_2 + 1)(n_2 + 2) C_{n_1+1, n_2+1}^2. \end{aligned} \quad (8)$$

Numerical calculations with using Eqs.(3),(8) reveals that there is no entanglement in the considered case in accordance with that for one-photon and degenerate two-photon processes.

4. Let consider the atom-atom entanglement when the initial atomic state is mixed so that the initial density operator are

$$\rho_A(0) = \prod_{i=1,2} [\lambda | + \rangle_i \langle + | + (1 - \lambda) | - \rangle_i \langle - |, \quad (9)$$

where  $\lambda/(1 - \lambda) = \exp(\omega_0/k_B T)$ . M.Kim et al.[3] and L.Zhou et al. [5] have shown that, with even a very small amount of mixture  $\lambda$ , atomic entanglement is washed out both for one-photon and degenerate two-photon interaction. Combining the results for three previous cases one can calculate the entanglement for initial mixed atomic state. The time dependence of  $\varepsilon$  for model with  $\lambda = 0.01$  (solid line) and  $\lambda = 0.05$  (dashed line) has presented in Fig.3 for  $\bar{n}_1 = \bar{n}_2 = 1$ . Note, that  $\lambda = 0$  corresponds to the atoms in the ground state. We find that with  $\lambda$  increasing further to 0.09, the entanglement is vanished.

So, we have shown that two identical atoms can be entangled through nonlinear nondegenerate two-photon interaction with two-mode thermal field with the exception of the case when both atoms are excited. For certain initial pure atomic states the two-atom entanglement may be stronger than that produced in one-photon and degenerate two-photon processes. For atoms are initially prepared in a thermal mixture of the ground and excited states the entanglement is greatly decreases with increasing the weight of the excited state in the mixture.

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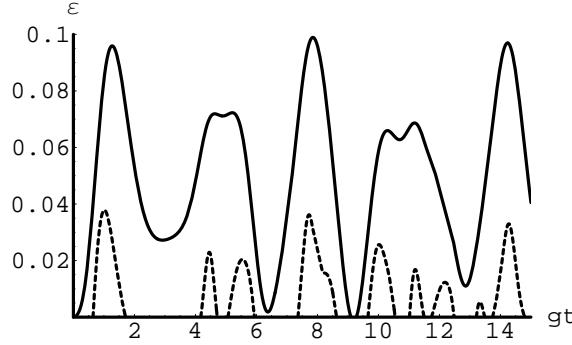


FIG. 1: Atom-atom entanglement induced by interaction with two-mode thermal field when the atoms are initially prepared in  $| +, - \rangle$  for  $\bar{n}_1 = \bar{n}_2 = 0.3$  (dashed line) and  $\bar{n}_1 = \bar{n}_2 = 1$  (solid line).

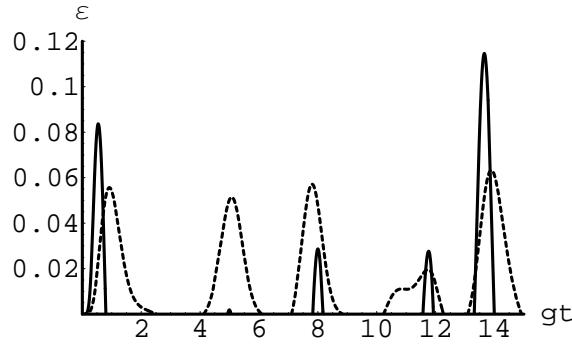


FIG. 2: Atom-atom entanglement induced by interaction with two-mode thermal field when the atoms are initially prepared in  $| -, - \rangle$  for  $\bar{n}_1 = \bar{n}_2 = 0.3$  (dashed line) and  $\bar{n}_1 = \bar{n}_2 = 1$  (solid line).

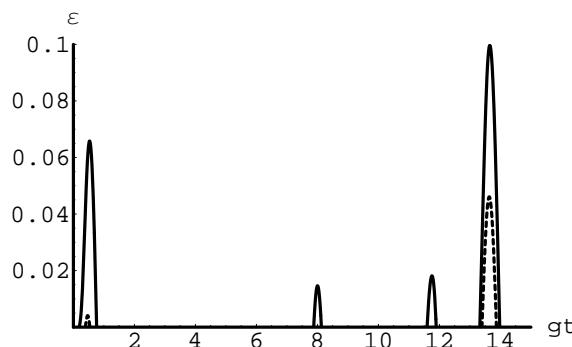


FIG. 3: Atom-atom entanglement induced by interaction with two-mode thermal field when the atoms are initially prepared in mixed state with  $\lambda = 0.01$  (solid line) and  $\lambda = 0.05$  (dashed line) for  $\bar{n}_1 = \bar{n}_2 = 1$ .